TRINITY COLLEGE

Year 12 Methods Unit 3,4
Test 6 2021

Calculator Assumed

Sampling

STUDENT'S NAME SOLNS

DATE: Monday 30 August **TIME:** 50 minutes **MARKS**: 49

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (12 marks)

A survey in Perth for high school students was conducted on ownership of a mobile phone. Out of 1845 students the survey found 1271 students owned a phone which was less than 1 year old.

Determine the following.

(a) (i) The sample proportion of high school students who own a mobile phone less than 1 year old. [1]

$\frac{1271}{1845}$

(ii) A 90% confidence interval for the proportion of Perth high school students who owned a mobile phone less than 1 year old. [2]

(0.6712, 0.7066)

(ii) What assumption was made in calculating this interval? [1]

APPROXIMATES A NORMAL DISTRIBUTION

(b) The margin of error in the calculated confidence interval. [2]

0.0177

Another three surveys of Perth high school students were conducted on ownership of a mobile phone less than 1 year old.

Survey 2	1635 out of 2338 high school students
Survey 3	840 out of 1298 high school students
Survey 4	2076 out of 3026 high school students

(c) Which of these surveys were more likely to have been taken outside of Perth? Justify your answer. [3]

2. 0.6993

3. 0.6471 SURVEY 3 OUTSIDE CI

4. 0.6861

(d) Using the sample proportion from the initial survey, calculate the sample size that will halve the margin of error for the proportion of Perth high school students who have a mobile phone less than 1 year old. [3]

$$\frac{0.0177}{2} = 1.645\sqrt{\frac{\frac{1271}{1845}\left(1-\frac{1271}{1845}\right)}{n}}$$

$$n = 7380$$

2. (3 marks)

A 99% confidence interval for a population proportion based on a sample size of 300 has a width d. What sample size is required to obtain a 99% confidence interval of width $\frac{d}{4}$?

$$ME = z\sqrt{\frac{p(1-p)}{n}}$$

$$\frac{d}{2} = z\sqrt{\frac{p(1-p)}{300}}$$

$$d = 2z\sqrt{\frac{p(1-p)}{300}}$$

$$\frac{d}{8} = z\sqrt{\frac{p(1-p)}{n}}$$

$$\frac{\cancel{2}\cancel{z}\sqrt{\frac{p(1-p)}{300}}}{\cancel{8}\,4} = \cancel{z}\sqrt{\frac{p(1-p)}{n}}$$

$$\frac{p(1-p)}{300 \times 16} = \frac{p(1-p)}{n}$$

$$n = 4800$$

3. (12 marks)

The Talsla Car Company produces electric cars. It wants to gather information consumers' interest in electric cars.

(a) In each of the following cases, comment, giving reasons, whether or not the proposed sampling method introduces bias.

(i) A Talsla Company representative stood near the Talsla display at a Car expo and asked people walking by "if they were to purchase an electric car, would it be a Talsla or an inferior brand"? [2]

BIAS eg LEADING QUESTION
CONVENIENCE SAMPLING

(ii) Two thousand randomly chosen mobile phone numbers were called and people asked "which brand of electric car would you prefer to buy"? [2]

BIAS eg NEED A MOBILE PHONE

One common problem with a particular electric car is the battery overheating. The manufacturer has found that 1% of their cars will suffer from this defect in the first two years. A sample of 250 is taken. Let the random variable X be the number of cars with overheating battery problems in the first two years in the sample of 250.

(b) What is the distribution of X? [2]

$X \sim b(250, 0.01)$

(c) What is the probability that more than four cars will have overheating battery problems within two years? [2]

$b(5 \le x \le 250, 250, 0.01)$

$= 0.1078$

In the random sample of 250 cars, three of them had battery over heating problems within the first two years.

(d) Calculate an approximate 94% confidence interval for the proportion of cars that have battery over heating problems within the first two years. [2]

(−0.001, 0.025)

(e) The company's quality control department wants the proportion of cars with battery problems to be between 0.5% and 1%. Based on your confidence interval, decide whether the quality control department is meeting its target. Justify your answer. [2]

LOWER END OF CI OK

UPPER END OF CI ABOVE 1%

∴ NOT MEETING TARGET

(f) Give an interpretation of the meaning of the 94% confidence interval. [1]

EXPECT 94% OF ALL C.I. WILL CONTAIN POPULATION PROPORTION.

4. (11 marks)

A beekeeper takes a sample of 290 bees in a beehive and determines that 34 of them are male drone bees.

(a) Determine a 90% confidence interval for the true proportion of drone bees in the hive. [2]

$(0.0862, 0.1483)$

It is an accepted fact that healthy beehives have a 0.15 proportion of drone bees.

(b) Determine whether the beehive is likely to be healthy. Justify your answer. [2]

NOT INSIDE 90% CI HOWEVER WOULD BE INSIDE BOTH 95% AND 99% C.I.

∴ LIKELY

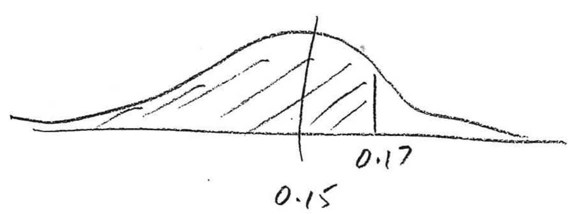
(c) The beekeeper decides to take another sample.

(i) If a new sample of 320 bees is taken, and knowing the true proportion of drones is 0.15, what is the probability that the sample proportion is at most 0.17? [3]

$p = 0.15$

$\sigma = \sqrt{\dfrac{0.15 \times 0.85}{320}}$

$= 0.01996$



$P(X \leq 0.17)$

$= 0.8418$

(ii) If a larger sample is taken, will the probability in (i) increase or decrease. Explain your answer. [2]

INCREASE , SMALL STANDARD DEVIATION

(iii) If the margin of error determined from a 90% confidence interval is to remain unchanged, how will the value of n, the sample size, change for a 99% confidence interval? Explain your answer. [2]

n LARGER , STANDARD DEVIATION NEEDS TO BE SMALLER

5. (11 marks)

The proportion of boats on the road being towed by vehicles that have the incorrect towing capacity is p.

(a) Show, using calculus, that to maximise the margin of error a value of $\hat{p}=0.5$ should be used. Note: as z and n are constants, the standard error formula can be reduced to $E=\sqrt{\hat{p}(1-\hat{p})}$. [4]

$$E = (\hat{p}-\hat{p}^2)^{\frac{1}{2}}$$

$$\frac{dE}{d\hat{p}} = \frac{1}{2}(\hat{p}-\hat{p}^2)^{-\frac{1}{2}}(1-2\hat{p})$$

$$\frac{1}{2}(\hat{p}-\hat{p}^2)^{-\frac{1}{2}}(1-2\hat{p}) = 0$$

$$(1-2\hat{p}) = 0$$

$$\hat{p} = \frac{1}{2}$$

$$\left.\frac{d^2E}{d\hat{p}^2}\right|_{\hat{p}=\frac{1}{2}} = -2 \quad \therefore \text{MAX}$$

(b) A consulting firm wants to determine p within 6% and with 99% confidence. How many towing vehicles should be tested at a random check? [3]

USE $\hat{p} = 0.5$ $\quad ME = 0.06$

$n = 461$

(c) Another random check sampling towing vehicles is made three months later. A 99% confidence interval calculated for the proportion of vehicles with incorrect towing capacity is (0.269, 0.373). Determine the number of vehicles in the sample that have an incorrect towing capacity. [4]

$$ME = \frac{0.373-0.269}{2} = 0.052$$

$$\hat{p} = \frac{0.373+0.269}{2} = 0.321$$

$$ME = Z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$n = 535$

0.321×535

$= 172$